M E M O R A N D U M

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SUBJECT: Poisson Point Process for Lake Okeechobee Minimum Flow and Level exceedance and violation events

EXECUTIVE SUMMARY

Lake Okeechobee (LOK) is the heart of water sources in South Florida. It is a vital ecosystem for wildlife, a recreational spot for tourism, a water supply storage facility and most importantly a buffer storage for flood protection. Extreme highs or extreme lows of water levels (stages) have significant adverse effects on the LOK functions including a serious threat to its levee, Herbert Hoover Dike (HHD). The recent rehabilitation of the HHD has forced water managers to change the lake management to a regulation schedule geared towards lower stages. Florida law (Chapter 373.042, F.S.) requires the state water management districts or the Department of Environmental Protection to establish minimum flows and levels (MFLs) for aquifers, surface watercourses, and other surface water bodies to identify the limit at which further withdrawals would be significantly harmful to the water resources or ecology of the area. Rivers, streams, estuaries and springs require minimum flows, while minimum levels are developed for lakes, wetlands and aquifers. SFWMD established an MFL for Lake Okeechobee in 2001. The objective of this study is to determine the likelihood of the Lake Okeechobee MFL being violated in the next 20 years, as required by Chapter 373, F.S. to be included in the Lower East Coast Water Supply Plan.

Lake Okeechobee stage, subject to rainfall and other stresses between 1965-2016 (52 years), is modeled under three different regulation schedules using South Florida Water Management District Regional Simulation Model (RSM). For each scenario, exceedances and violations are defined and represented using Poisson Point Process where waiting time between events were fitted to exponential distributions. Monte Carlo

simulation of exceedance and violation events in the next 52 years yield comparable statistics of the original time series being modeled by the RSM. Monte Carlo simulation in the next twenty years show that under the old schedule a maximum of two events would occur with 90% probability while under the new schedules there is 70% probability of three or more events would occur in the same period.

Poisson Point Process for Lake Okeechobee Minimum Flow and Level exceedance and violation events

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Introduction

Lake Okeechobee (LOK) is the largest natural freshwater lake in the Southeastern United States with a surface area of 730 square miles and average depth of 9 feet. LOK is considered the heart of the water resources system in south Florida. The lake provides natural habitat for wildlife, attracts recreation enthusiast from around the world, serves as the primary water supply storage for South Florida, and most importantly it provides flood protection in a such a low topographic system. To maintain healthy and safe conditions the lake has been operated by regulation schedules that have evolved over the years geared towards optimal performance to achieve certain goals while meeting certain legal requirements. Following Hurricane Katrina in 2005, there was an increased concern about LOK experiencing above average water levels which caused stress to the structural integrity of the Herbert Hoover Dike (HHD) that surrounds the lake, as well as the lake's natural habitat. In 2008 the HHD rehabilitation started along with implementation of an interim regulation schedule (LORS08) superseding the Water Supply and Environment (WSE) schedule and being geared toward maintaining significantly lower water levels. Regulation schedules geared toward better performance of one objective often reduces the ability to satisfy other competing objectives. For example, the minimization of high lake stage events comes on the expense of increasing the frequency of low stage events. When water levels reach extreme low stages, there is always the potential for negative impacts to water supply, ecosystem, and navigation. Of interest in this study is to investigate the extreme low water levels as a biproduct of the regulation schedule under consideration.

A Minimum Flows and Minimum Water Levels (MFLs) program administered by the District to define extremely low flows and water levels as an important step in the District's work of planning for adequate water supplies while also protecting water resources from significant harm. Minimum levels have been established for lakes, wetlands and aquifers in south Florida. Minimum flows have been set for rivers, streams, and estuaries. MFLs are defined as the minimum flows or minimum water levels, adopted by the District Governing Board pursuant to Sections 373.042 and 373.0421, Florida Statutes, at which further withdrawals would be significantly harmful to the water resources or ecology of the area. Subsection 40E-8.221(1), Florida Administrative Code, F.A.C, defines an MFL exceedance for Lake Okeechobee as the decline below 11 feet NGVD for more than 80, non-consecutive or consecutive, days, during an 18-month period. The 18-month period shall be initiated following the first day Lake Okeechobee falls below 11 feet NGVD, and shall not include more than one wet season, defined as May 31st through October 31st of any given calendar year. An MFL violation occurs in Lake Okeechobee when an exceedance, as defined herein, occurs more than once every six years.

Of interest to the planner is to evaluate the likelihood of MFL exceedances and violations through an extended time horizon (e.g., 20 years) under several Lake Okeechobee management scenarios. Traditionally, future climatic data are represented by an ensemble of synthetic climatic data generated based on climatic historical data. Each synthetic climatic data scenario is input to the physically based computer model to simulated stage response. Exceedance and Violations are then recorded for each model run. This step is cumbersome and computationally intensive making the quantification of future exceedance and violation events difficult. In this study we attempt to directly model the exceedance and violation events for Lake Okeechobee as a Temporal Point Process (TPP) based on historical data.

A TPP often represents a stochastic process of a binary event timeseries on a continuous time axis (Daley and Vere-Jones, 2003). It is used to describe data that are localized at a finite set of time points. Unlike continuous-valued processes, TPP is binary in nature where it takes on only one of two possible values at a given time, indicating whether an event occurs at that time. Figure 1 depicts highly irregular timeseries of stage data (beyond a deterministically determined average seasonality). Exceedance and Violation events, as defined above, are driven by climatic uncertainty as well as complex operational protocols. Such a collection of events occurs randomly at a discrete set of points along the time axis and can be modeled using TPP theory. The points arise from a random process, described by the local intensity $\lambda(s)$, which measures the expected density of points at a given location, s, in space (or time). If points arise independently and at random, the local intensity can be described by a homogenous Poisson distribution and is referred to as a Poisson Point Process (Cox and Isham, 1980). If event locations are independent but the intensity varies spatially, nonstationary process, the distribution arises from an *inhomogenous point process* (i.e. $\lambda(s)$ varies). The latter is also called *inhomogenous Poisson process*. Other PP studies in the literature of hydrology, including Renewal Processes, Markov Chains, Poisson Processes, and other point processes (Green, 1964; Katz, 1977; Richardson,1981; Smith and Karr, 1983; Foufoula-Georgiou and Lettenmaier, 1987; Rodriguez-Iturbe et al., 1988; Cowpertwait et al., 1996; Ali and Lall, 1998, Wilks and Wilby, 1999). These studies include weather forecasting (e.g., hurricane), stochastic weather generation, climate impact assessment, climate model downscaling, hydrological modeling, ecological modeling, agricultural modeling, and subsurface geologic modeling.

In this planning study we model the exceedance and violation events (defined above and depicted in Figure 1) using Poisson Process. Traditionally we define a TPP $\boldsymbol{N}$ as a random measure of a certain event (exceedance or violation) along the time axis $\boldsymbol{T}$, taking values in the non-negative integers $\boldsymbol{Z}^+$ (or infinity). In this framework the measure $\boldsymbol{N(t)}$ represents the number of events falling in the subset $\boldsymbol{t}$ of $\boldsymbol{T}$. For the purpose of this problem our attention is restricted to the case where $\boldsymbol{N}$ may contain only a finite number of points on any bounded subset $\boldsymbol{t}$ of $\boldsymbol{T}$ (Jacod, 1975, Bremaud, 1981, and Anderson et. al., 1993). Detailed presentation of Poisson Process Modeling is provided below.

Poisson Point Process, *PPP*

Poisson Point Process is the most used TPP in stochastic modeling (Cox and Isham 1980), which is a simple point process ***N*** such that the number of points in any subset follows a Poisson distribution and the numbers of points in disjoint subsets are independent. That is, *N* is a Poisson process if $N(t_1), ..., N(t_k)$ are independent Poisson random variables, for any disjoint, measurable subsets $t_1, ..., t_k$ of *S*. The behavior of a simple PPP *N* is typically modeled by specifying its *conditional intensity*, λ(t), which represents the infinitesimal rate at which events are expected to occur around a particular time *t*, conditional on the prior history of the point process prior to time *t.* A formal definition of the intensity function of a simple PPP is expressed as

$$\lambda(t) = \lim_{\Delta t \to 0} E\{N[t, t + \Delta t)] \mid Ht\} / \Delta t, \quad (1)$$

A stationary (homogeneous) Poisson process has a constant conditional rate: λ(t), = α for all *t*. which is the expected value of event occurrence. Assuming events are independently and identically distributed.

Exponential Distribution of the inter-event times.

This classic definition of the temporal point process is not the most intuitive means of characterization. Another way of characterizing **TPP** is to consider the times of events occurring between time 0 and time ***S****.* One may characterize *N* as an ordered list $\{s_1, ..., s_n\}$ of event times. One may alternatively convey the equivalent information about *N* via the inter-event (waiting) times $\{u_1, ..., u_n\}$, where $u_i = s_i - s_{i-1}$, with the convention that $s_0 = 0$. A TPP ***N*** may alternatively be characterized by the waiting time at any time ***t*** between 0 and ***S.*** That is, N(t, u)|0→S where the variable is now the waiting time between events (rather than the number events per an arbitrary time interval), and is modeled using exponential distribution (rather than Poisson distribution). It should be noted that number of occurrences ***N(t)*** follows a Poisson Process, the waiting time between these events **u** is exponentially distributed. With λ being the intensity of the rate of arrival and is expressed as

$$\lambda(t) = \lim_{u \to 0} P\{N[t, t + u)] > 1 \mid u\} / u \quad (2)$$

In this study we use a simple homogenous Poisson Point Process to model the exceedance and violation events by modeling the inter-event time as the random variable according to the following exponential distribution

$$p(t) = \lambda e^{-\lambda t}$$

Lake Okeechobee Regulation Schedules

The District has undergone three main regulation schedules since 2000. Water Supply and Environment, (WSE) utilized in operations from 2000-2007, followed by the Lake Okeechobee Regulation Schedule 2008, (LORS08), utilized in operations from 2008-2023, which precede the Lake Okeechobee System Operating Manual, (LOSOM), a proposed schedule awaiting authorization. WSE is pre-HHD rehabilitation, LORS08 was during HHD rehabilitation and LOSOM is post HHD rehabilitation. Hydrologic simulations of LOK stage under those schedules are based on the Regional Simulation Model (RSM) *LORS final EIS 2007 and LOSOM final EIS 2024.* For MFL evaluation, three schedules of LORS08 were considered: 1) The WSE, 2) The "current" condition representing LORS08 operations circa 2019 at the start of the project (ECB19) and 3) LOSOM based preferred alternative for a future 2025 (PA25). Please note that the Period of Record (POR) for the simulation data is 1965-2016.

Lake Okeechobee regulation schedules have evolved over the past 24 years to accommodate the HHD rehabilitation. Prior to HHD, LOK was operated under WSE regulation schedule that promoted higher storage hence higher structural risk against HHD. During the rehabilitation LOK was operated under LORS08 (ECB19), an interim schedule, which resulted in lower storage. With the completion of the HHD rehabilitation a new regulation schedule LOSOM (PA25) is to be deployed which brings the LOK stage to higher levels but not as high as of those of WSE due to the environmental releases to Caloosahatchee estuaries and Everglades.

Procedures to simulate exceedance/violation events

- For each of the above-mentioned scenarios (WSE, ECB19, and PA25), run RSM model for the period of record of available input data (1965-2016).
- Calculate the exceedance and violation events from the LOK stage output time series as defined above.

- Fit exponential distribution (with one parameter λ) for the waiting time between events for "exceedances" and "violations". λ Is the average of the event waiting time.
- Using Monte Carlo (MC) technique to simulate event sequence for the next 20 years by sampling the waiting time between events. Sampling is a random selection from the respective distribution allowing for λ to randomly change between its confidence band with normal distribution.
- Repeat the experiments for 1000 times.
- Obtain 5%, 50% and 95% percentiles and verify against original data.
- Perform a histogram comparative analysis for the 4 alternatives.

Results

The above procedures were applied to WSE, ECB19, and PA25 schedule scenarios. RSM LOK time series are used to derive exceedance and violation events as defined above. Exceedance and Violation histograms of the waiting time are presented in Fig. 2. The scenarios' waiting time (exceedance or violation) are skewed to the left (except for zero violations). A summary table of the observed exceedance and violation waiting times are given in Table 1. Figure 2 and Table 1 show that WSE data exhibits longer waiting time for exceedance and no violation events, PA25 shows shorter waiting time for exceedance and violation events while ECB19 shows something in between. This is consistent with the progression of the HHD rehabilitation where WSE promoted higher stage. During the rehabilitation period, ECB19 lowered the stage substantially; meanwhile, PA25 raised the stage, but not to WSE extent. Exponential probability density function (pdf) was fitted to each data set (waiting times). Note that parameter λ is the mean value (columns 1 and 4 of Table 1 for exceedance and violation respectively.

Waiting times between the exceedance and violation events of TPP were simulated according to the above procedures Before we employ the above procedures for application, we perform two tests. The first test is to evaluate the results sensitivity to sample size. Figure 3 depicts the exceedance and violation histograms for the next 20 years, for 100, 500, 1000 and 10,000 realizations. All graphs show that results are robust, and they stabilize for sample size greater than 100. We choose sample size of 1000 for our subsequent analysis. The second test is to verify that the statistics of the simulated events for 52 years (same as the historical POR) encompass the corresponding historical statistics. Table 2. shows 5th, 50th, and 95th percentiles for 1000 simulations of 52 years POR, and the historical means of the historical exceedance and violation events for WSE, ECB19 and PA25 respectively. Results show that simulated statistics capture the historical means properly. Figure 4 depicts event histograms simulated for 20 years for the three scenarios. Exceedance histograms show left skewed distribution for WSE with 75% probability of maximum 3 events while the other two scenarios more than 80% probability of occurrence of more than or equal three events. Violation histograms show 80% chance of maximum of 2 events for WSE while ECB19 and PA25 show more than 75% of at least 2 events.

Figure 5 depicts multiple histograms across the 3 scenarios for exceedance and violations. This figure clearly shows three distinct patterns serving three different purposes. WSE scenario there is less than 20% chance exceedance exceed 3 events in the next 20 years. The ECB19 and PA25 scenarios have 85% chance of exceedance of three or more events with PA25 scenario being less vulnerable. With regards to violations, WSE is 90% likely to have maximum of 2 events while ECB19 and PA25 have 75% chance to produce 2 or more violation events in the next 20 years with PA25 being less susceptible. Table 4 shows summary statistics for the 5%, 50% and 95% tiles for each of the three scenarios for both exceedance and violation events. The tabulated data show that WSE exhibits the least scenario of MFL exceedance and violation vulnerability followed by PA25 followed by ECB19.

Conclusion

Lake Okeechobee regulation schedules have evolved over the past 24 years to accommodate the HHD rehabilitation. WSE (prior to HHD), LORS08 (during HHD) and LOSOM (post HHD) regulation schedules have served different purposes hence resulting in different LOK low and high-water conditions. The objective of this study was to evaluate and compare compliance with MFL exceedance and violation criteria for these three scenarios. Exceedance and violation events were modeled as Poisson Point Process where waiting time between events follow an exponential distribution. Monte Carlo simulation technique performed showed that a sample size of 500 or more produced robust results. MC application to simulate 52 years of events substantially mimicked the historical events. Results consistently show that MFL exceedance and Violation were the lowest under the WSE scenario (prior to HHD) and the worst under ECB19 (during the HHD where LOK experienced lower water storage) while PA25 was slightly more improved than ECB19. MC application of Poisson Process to simulate the exceedance and violation events showed that WSE is most likely to produce 2 or less events while EXCB19 and PA25 are most likely to produce three or more events. While the conclusion here assumes stationary process, we recommend the incorporation of climate change (nonstationarity) through the use of inhomogeneous Poisson Process.

References

Jacod J. Multivariate point processes: predictable projections, Radon-Nikodym derivatives, representation of martingales. Z. Wahrsch. Verw. Gebiete 1975; 31: 235-253.

Brémaud P. *Point Processes and Queues: Martingale Dynamics.* 1981; Springer-Verlag, New York.

Andersen P, Børgan O, Gill R, Keiding N. *Statistical Models Based on Counting Processes.* 1993; Springer-Verlag, New York.

Green, J. R.: A model for rainfall occurrence, J. Roy. Stat. Soc. Ser. B, 26, 345–353, 1964.

Katz, R. W.: Precipitation as a chain-dependent process, J. Appl. Meteorol., 16, 671–676, 1977.

Richardson, C. W.: Stochastic simulation of daily precipitation, temperature, and solar radiation, Water Resour. Res., 17, 182–190, 1981

Smith, J. A. and Karr, A. F.: A point process model of summer season rainfall occurrences, Water Resour. Res., 19, 95–103, 1983.

Foufoula-Georgiou, E. and Lettenmaier, D. P.: A Markov renewal model for rainfall occurrences, Water Resour. Res., 23, 875–884,
1987.

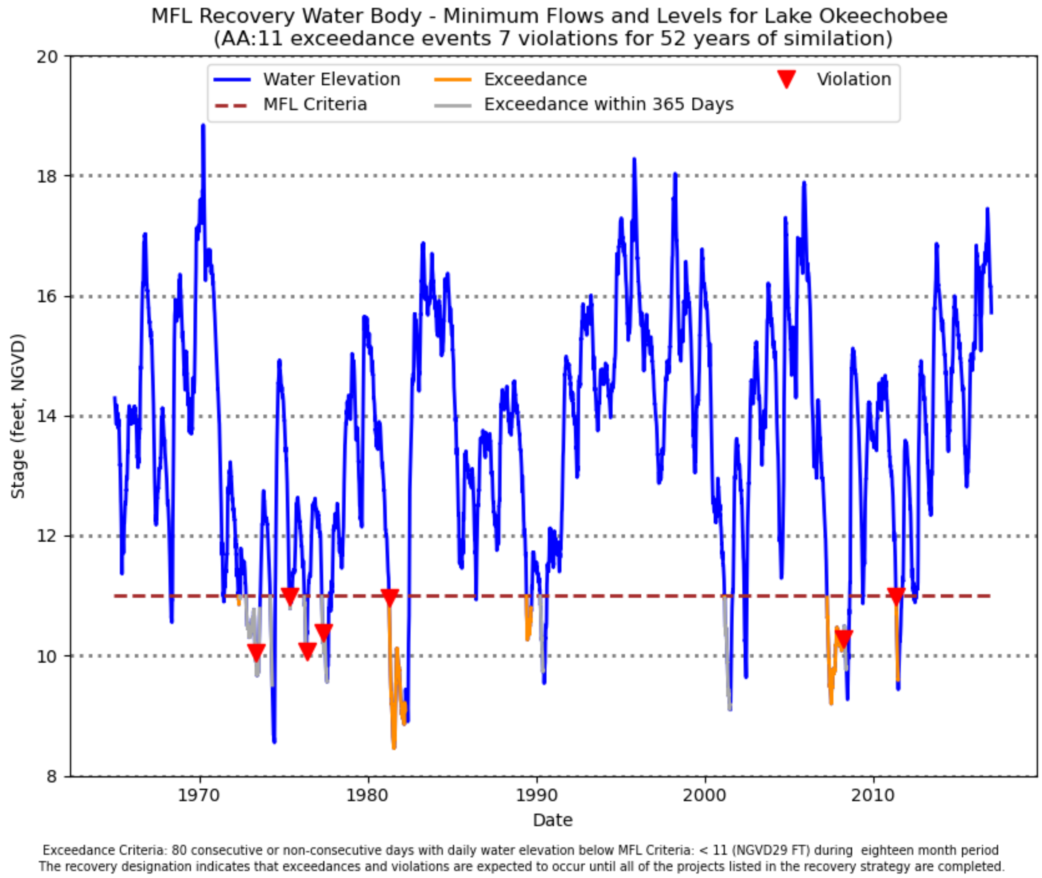
Cowpertwait, P., O'Connell, P., Metcalfe, A., and Mawdsley, J.: Stochastic point process modelling of rainfall. I. Single-site fitting and validation, J. Hydrol., 175, 17–46, 1996.

Rodriguez-Iturbe, I., Cox, D. R., and Isham, V.: A point process model for rainfall: further developments, P. Roy. Soc. Lond. A Mat., 417, 283–298, 1988.

Cox, D. R., and Isham, V., Point Processes, Chapman & Hall/CRC Monographs on Statistics and Applied Probability, 1980.

Ali, A., Lall, U. A Continuous Parameter Homogeneous Semi-markov Model For Stratigraphic Analyses From Borehole Data, *The Log Analyst* 39 (02), 1998.

Wilks, D. S. and Wilby, R. L.: The weather generation game: a review of stochastic weather models, Prog. Phys. Geog., 23, 329–357, 1999.



Figue 1. Lake Okeechobee historical stage and MFL exceedance and violation events.

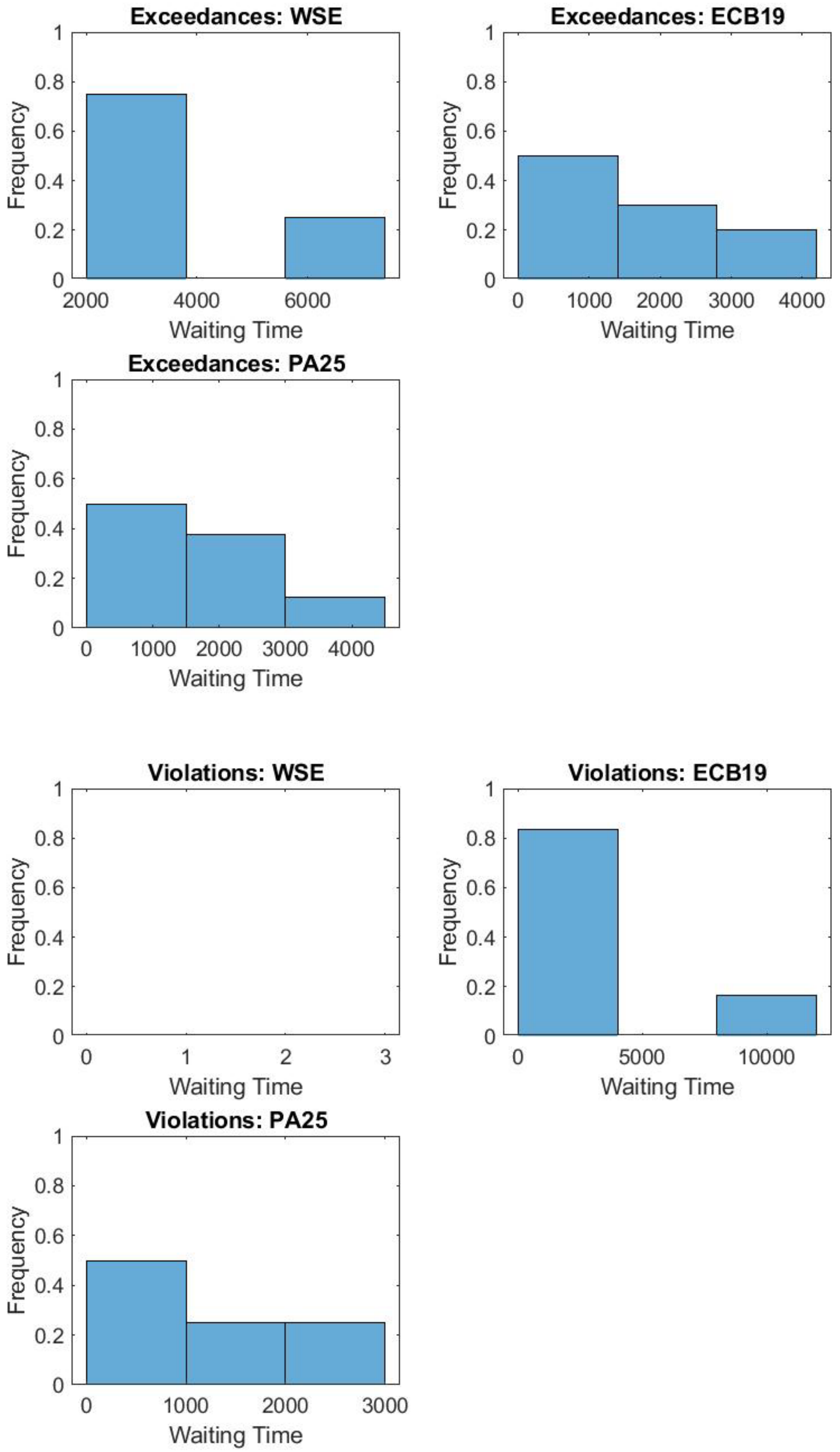


Figure 2. Exceedance and Violation histograms of historical waiting time.

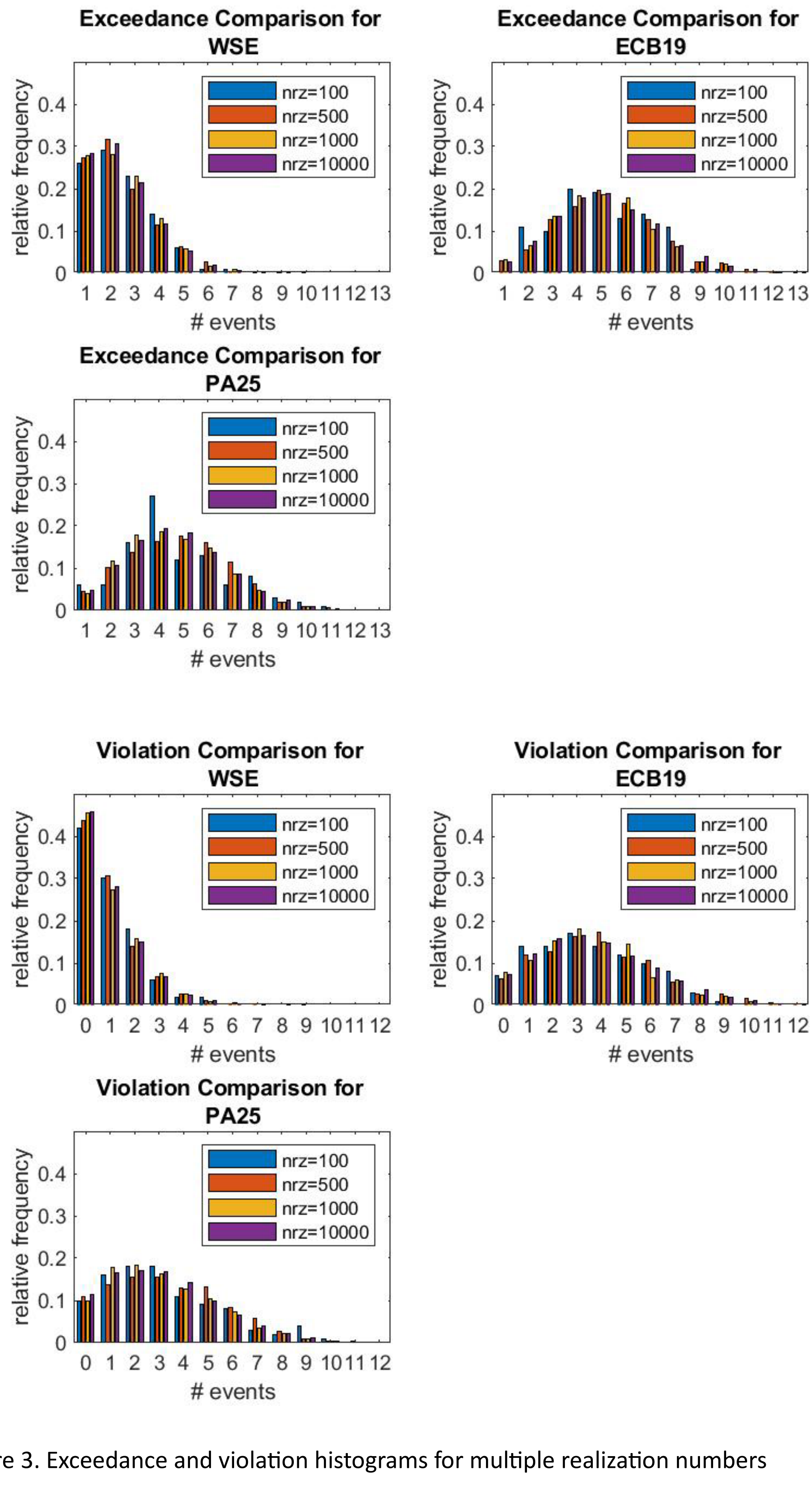


Figure 3. Exceedance and violation histograms for multiple realization numbers

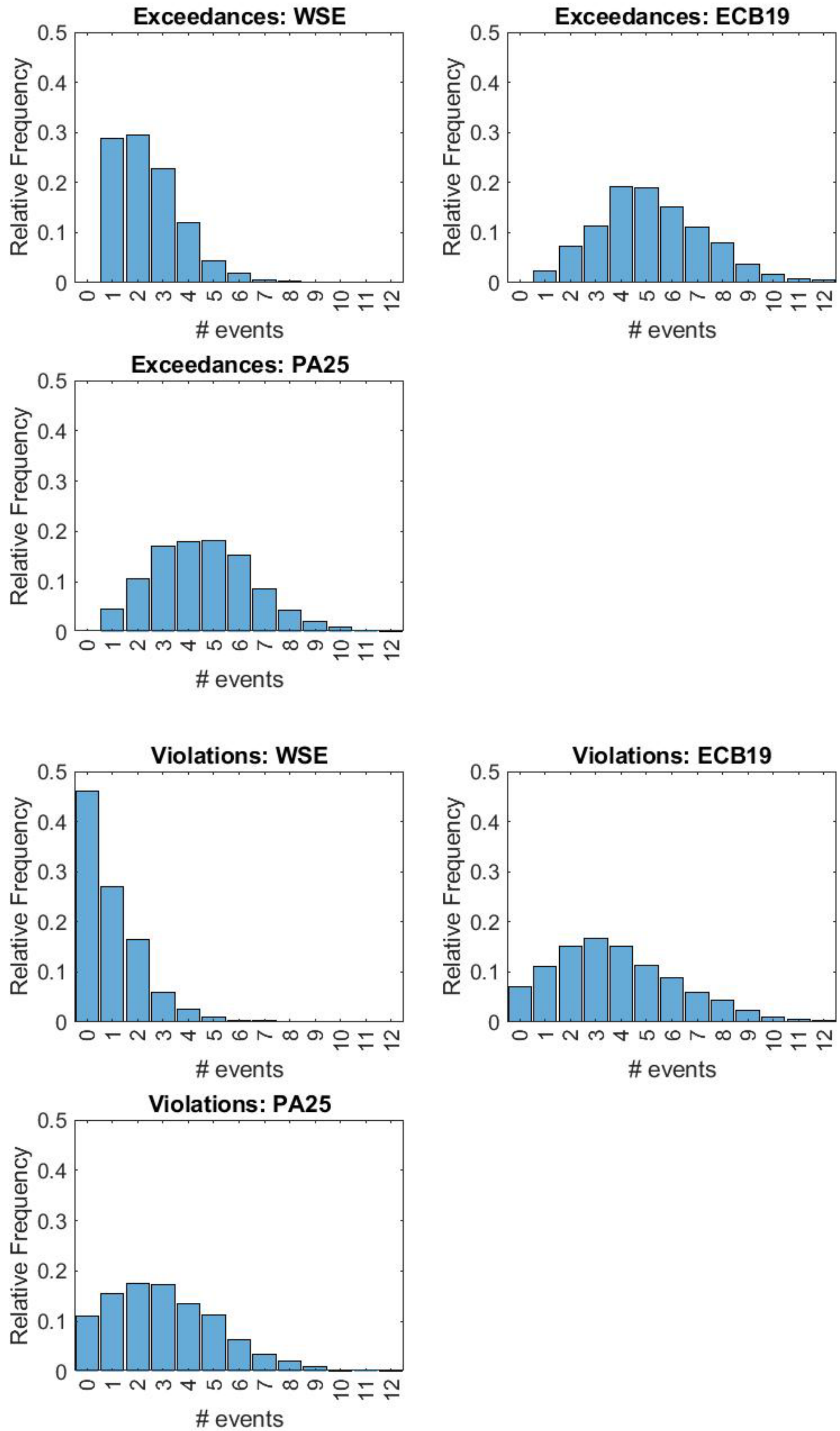


Figure 4. Exceedance and Violation histograms for LOK scenarios based on 1000 samples.

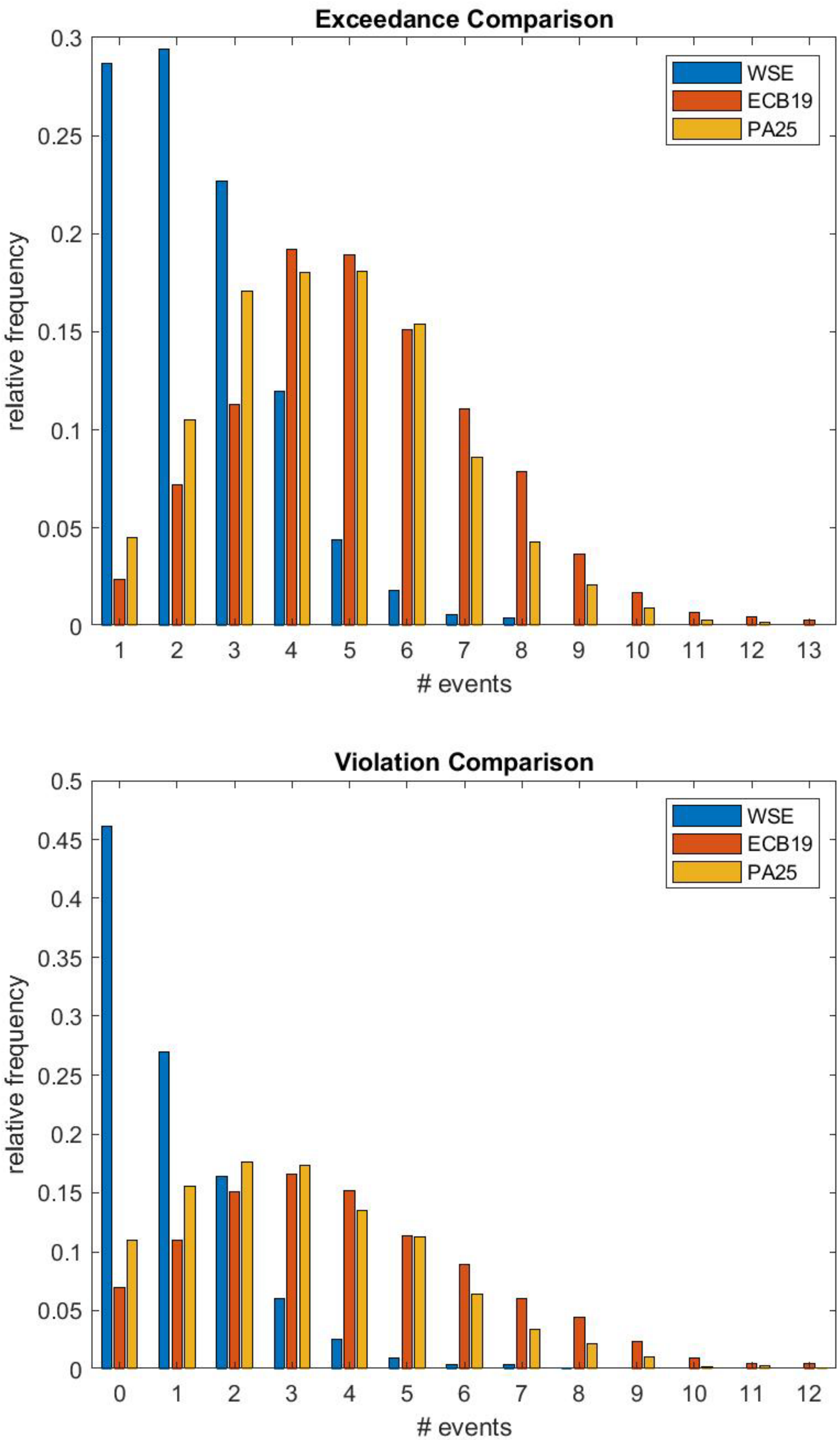


Figure 5. Exceedance and Violation comparisons across the four scenarios.

Table 1. Summary statistics for Historical Data of waiting time between events.

	Exceedance events				Violation events			
	Mean	Median	Std-Dev.	Skew-ness	Mean	Median	Std-Dev.	Skew-ness
WSE	3887	3019	2333	1.00	NA	NA	NA	NA
ECB19	1700	1477	1256	0.52	2699	1147	4084	1.63
PA25	1938	1814	1424	0.28	1153	787	1257	0.79

Table 2. Simulated events summary statistics and the corresponding historical number of events.

	**Exceedance events over next 52 years			Exceedance Data	**Violation events over next 52 years			Violation Data
	5%tile	Median	95%tile		5%tile	Median	95%tile	
WSE	1	4	8	**4**	0	1	5	**0**
ECB19	5	11	18	**10**	2	8	15	**6**
PA25	3	10	16	**8**	1	6	13	**4**

Table 3. Summary statistics for simulated events for 20 years based on1000 realizations.

	Exceedance events				Violation events			
	Mean	Median	Std-Dev.	Skew-ness	Mean	Median	Std-Dev.	Skew-ness
WSE	2.44	2	1.33	1.02	0.99	1	1.25	1.69
ECB19	5.22	5	2.16	0.50	3.83	4	2.48	0.64
PA25	4.60	4	2.01	0.43	3.09	3	2.19	0.69

Table 4. Simulated events summary statistics for the next 20 years.

	Exceedance			Violation		
	5%tile	Median	95%tile	5%tile	Median	95%tile
WSE	1	2	5	0	1	3
ECB19	2	5	9	0	4	8
PA25	2	4	8	0	3	7